Lecture 8 – 09/04/2025

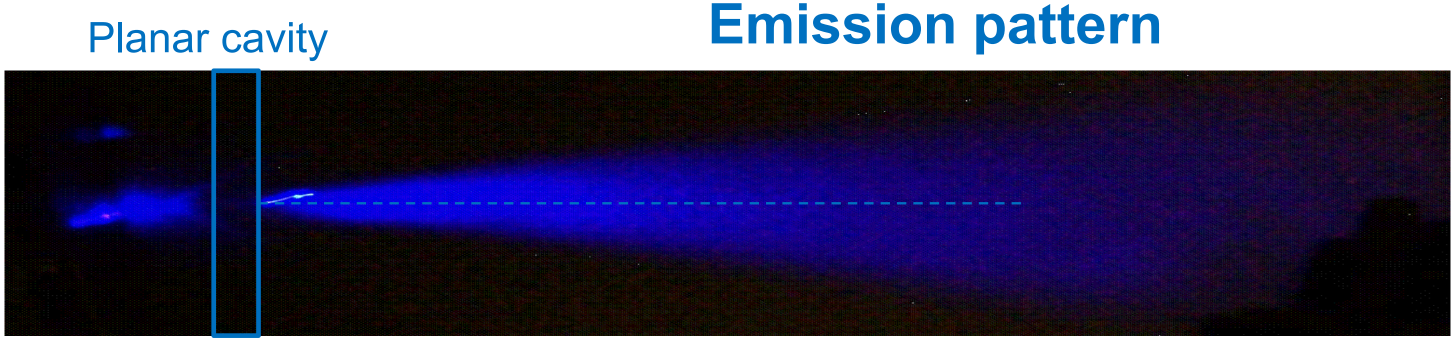
Insights into the weak coupling regime: the Purcell effect

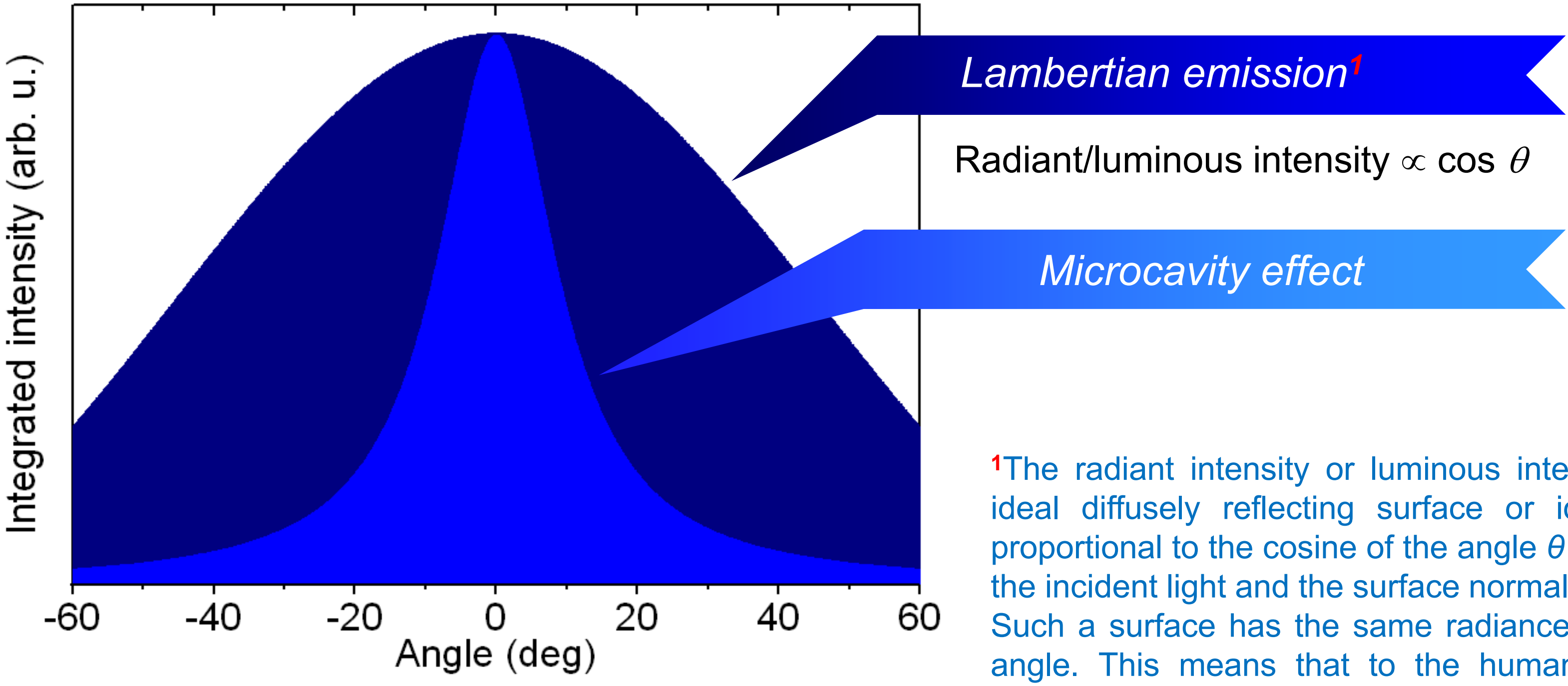
Light-emitting diodes: historical account

Light-matter interaction
Case of the Purcell effect

Weak coupling regime

Emission pattern



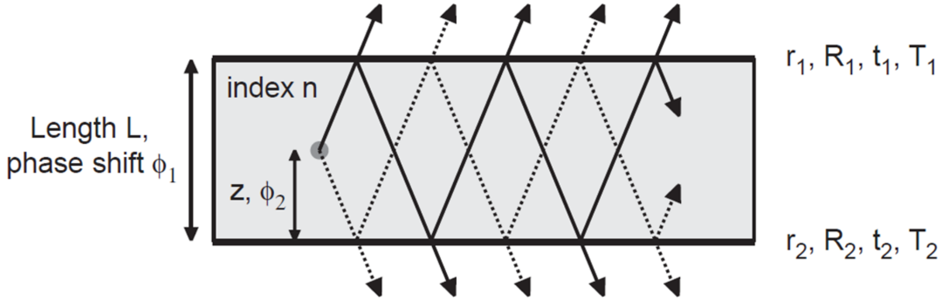


Case of a light source in resonance with the cavity mode

[1]The radiant intensity or luminous intensity observed from an ideal diffusely reflecting surface or ideal diffuse radiator is proportional to the cosine of the angle θ between the direction of the incident light and the surface normal.
Such a surface has the same radiance when viewed from any angle. This means that to the human eye it has the same apparent brightness (or luminance).

Weak coupling regime

Transmission for a point source inside a Fabry-Perot cavity



See, e.g., A. Kastler, Appl. Optics **1**, 17 (1962)

⚠ $r = \sqrt{R}$ and $t = \sqrt{T}$

Position-dependent part

$$T_{top} = \frac{T_{FP} \times \zeta\left(\phi_2^{eff}\right)}{T_2} = \frac{T_1\left|1 + r_2 e^{2i\phi_2^{eff}}\right|^2}{\left|1 - r_1 r_2 e^{2i\phi_1^{eff}}\right|^2} \text{ with } T_{FP} = \frac{T_1 T_2}{1 + R_1 R_2 - 2\sqrt{R_1 R_2}\, cos\left(2\phi_1^{eff}\right)}.$$

Transmission coefficient through the top mirror

Exaltation or inhibition due to the modal structure of the whole cavity (cf. slide 12, Lecture 7)

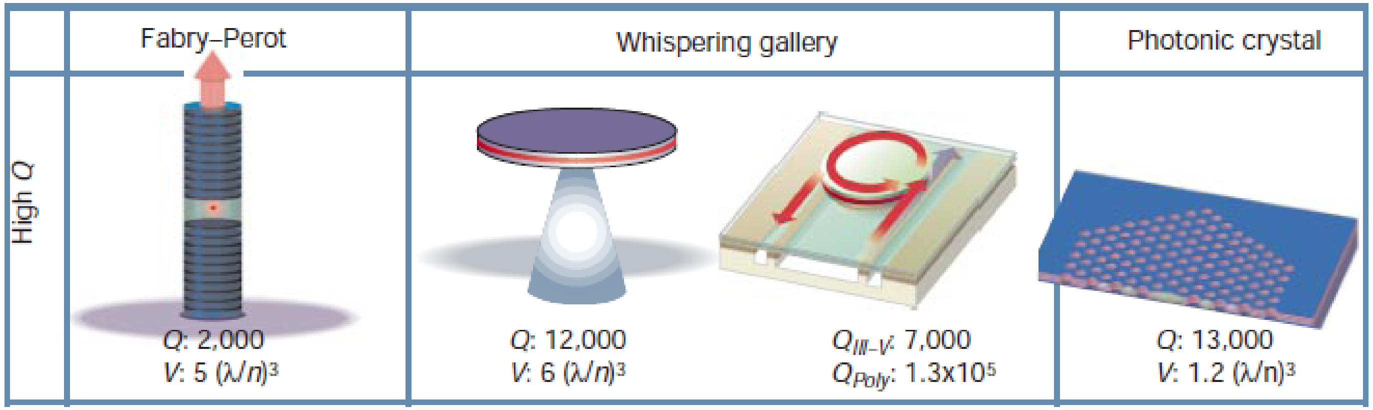
$$T_{top} = \frac{T_1\left(1 + R_2 + 2\sqrt{R_2}\, cos\left(2\phi_2^{eff}\right)\right)}{1 + R_1 R_2 - 2\sqrt{R_1 R_2}\, cos\left(2\phi_1^{eff}\right)} \text{ with } \phi_1 = kL\cos\theta \text{ and } \phi_2 = kz\cos\theta \text{ and } k = \frac{2\pi n}{\lambda}.$$

$$2\phi_1^{eff}(\lambda,\theta) = 2\phi_1 - arg(r_1) - arg(r_2) \text{ and } 2\phi_2^{eff}(\lambda,\theta) = 2\phi_2 - arg(r_2)$$

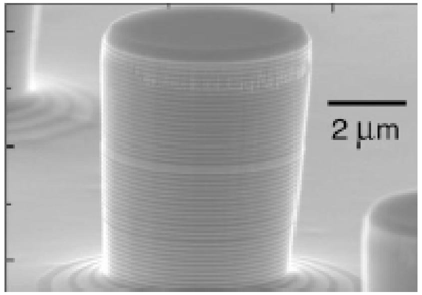
Effective phase shifts

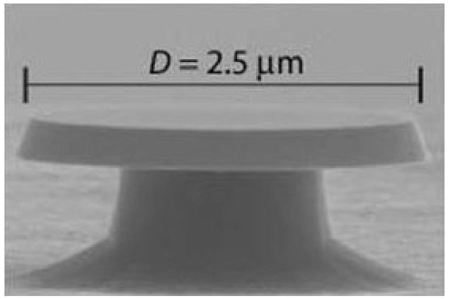
ζ is called the standing wave factor, which expresses the dependence of the emitted intensity on the position of the point source (transmission will be high in a particular direction if the source is located at an antinode of the standing wave field). It can vary from 0 up to 4 (if r_2 = 1).

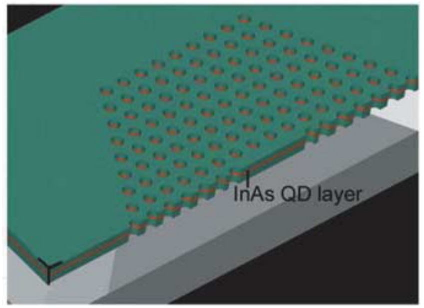
Cavities for Purcell effect studies



Q/V ratio: key figure of merit to observe the Purcell effect (quantity to be maximized)

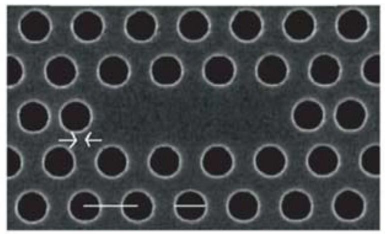






Investigation of peculiar photon emission regimes of solid-state emitters embedded in a resonator:

1. **The Purcell effect** (enhanced spontaneous emission rate)
2. **The strong coupling regime** (admixed light-matter eigenstates) $\Rightarrow$ not studied here!

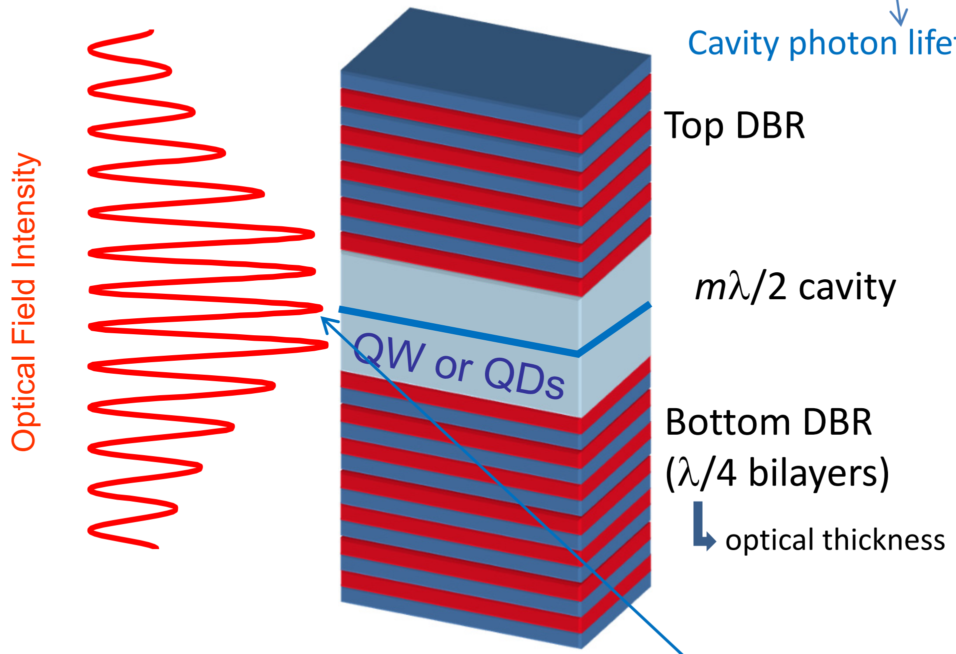


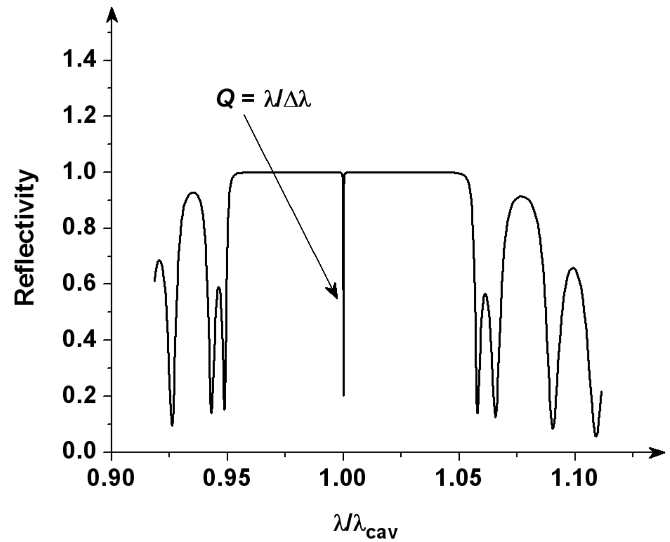
Semiconductor micropillars

1. **Growth of high-quality factor ($Q = \lambda_c/\Delta\lambda_c = \omega\tau_c$) epitaxial planar microcavities** ($m\lambda/2$ optical cavity ($m \in N^*$) surrounded by high-reflectivity distributed Bragg reflectors (DBRs))
2. **Etching process** (reactive ion etching (RIE) + electron beam lithography (EBL) or focused ion beam (FIB) etching) **of posts to obtain 3D optical confinement** ($\varnothing$ ~ 0.5-10 μm)

Cavity photon lifetime

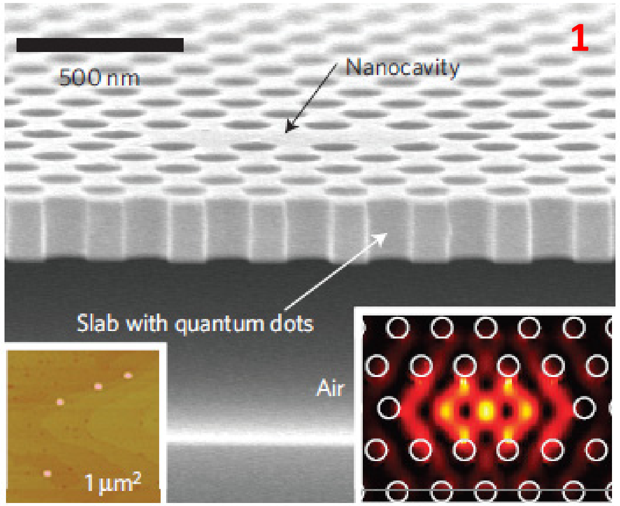
⇔ 1D photonic crystal





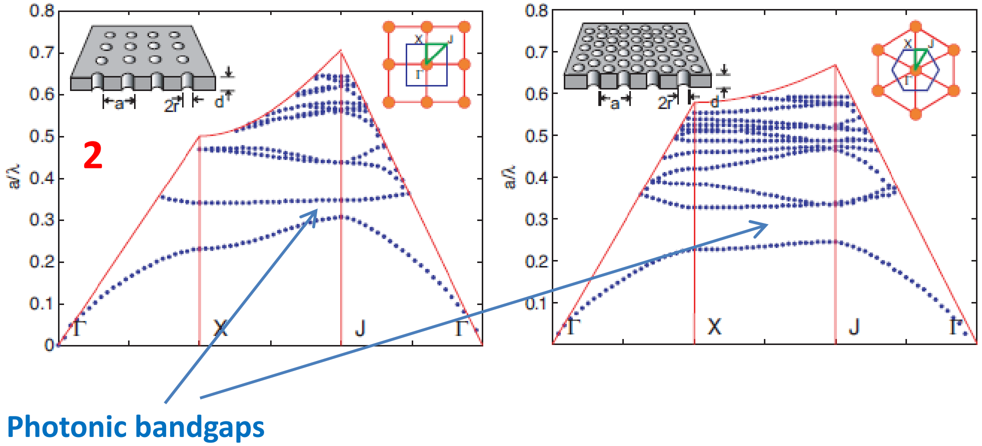
Standing electromagnetic wave along the growth axis (cavity mode) **with active layer** (QWs or QDs) **positioned at field antinodes** (enhanced light-matter interaction)

Two-dimensional photonic crystals (2D-PhCs)



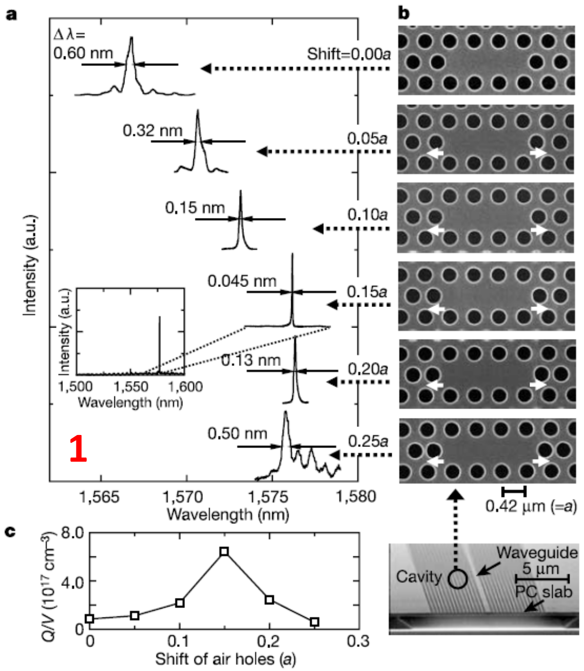
2D periodic refractive-index modulation (achieved by drilling a periodic array of air holes in a semiconductor slab by means of EBL) **which induces a photonic bandgap (PBG) effect in the plane of the membrane** (analogy with electronic bandgap of semiconductors)

The forbidden energy bands are the source of 2D-DBR confinement. PBGs are polarization sensitive (TE-TM modes do not experience the same bandgap)



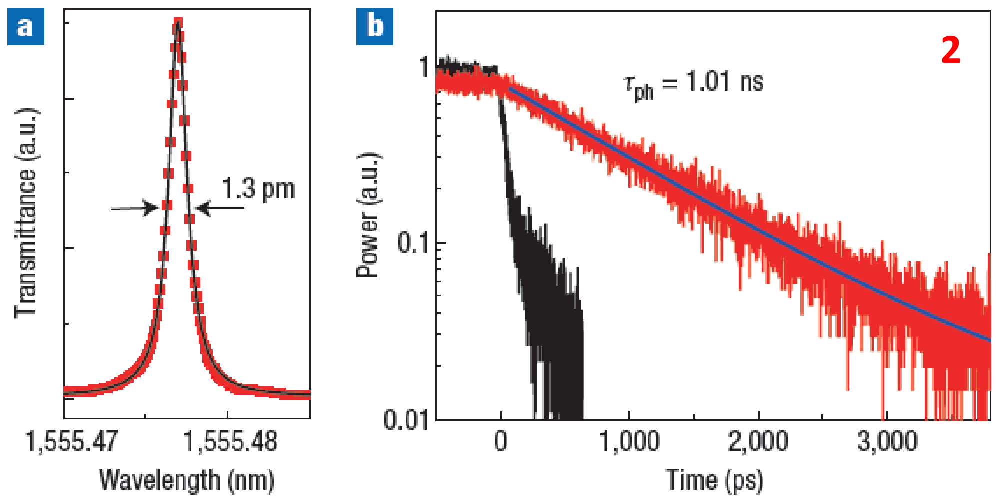
[1]M. Nomura *et al.*, Nat. Physics **6**, 279 (2010). (> 310 citations)
[2]D. Englund *et al.*, Opt. Express **13**, 5961 (2005). (> 170 citations)

Two-dimensional photonic crystals (2D-PhCs)



High Q factor small modal volume cavities achieved by suppressing holes
Positioning of surrounding holes highly critical to maximize Q factor
(most common design: modified L_3 cavity (3 missing holes) in a triangular 2D-PhC lattice)
Vertical emission

Ultra high Q (> 10^6) factor microcavities (silicon 2D-PhC slab)
Time-domain measurements leading to τ_c > 1 ns



[1]Y. Akahane *et al.*, Nature (London) **425**, 944 (2003). (> 2350 citations)
[2]T. Tanabe *et al.*, Nat. Photonics **1**, 49 (2007). (> 320 citations)

Automated optimization of PhC slab cavities

Genetic algorithm + 3D-FDTD or guided mode expansion (GME) calculations[1-2]

Powerful tool to compute:

- Quality factor (Q) of defect-cavities, e.g., L3, H0, H1
- Mode volume (V_m)

$$V_m = \frac{\int \varepsilon(\mathbf{r}) |\mathbf{E}(\mathbf{r})|^2 d^3 r}{\max\left[\varepsilon(\mathbf{r}) |\mathbf{E}(\mathbf{r})|^2\right]}$$

$$\frac{1}{Q_{exp}} = \frac{1}{Q_{th}} + \frac{1}{Q_{abs}} + \frac{1}{Q_{fab}}$$

$Q_{exp} > 1 \times 10^6$ within reach

Simple optimization strategy

- Small set of variational parameters, e.g., spatial shift of holes next to the defect ($\neq$ from empirical shift)

No need for a "global exploration of parameter space"

- Limited impact on V_m (e.g., restrictions apply to Q/V)

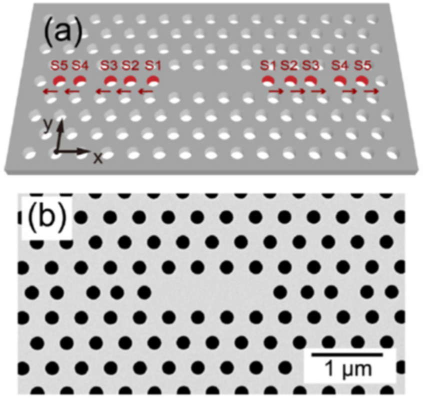
Main benefits:

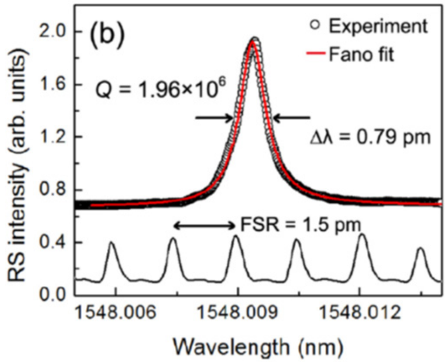
- Access to statistical analysis of fabrication imperfections that are plugged into the 3D-FDTD calculations ($\Rightarrow$ resilience of a given design to imperfections)

[1]M. Minkov and V. Savona, Sci. Rep. **4**, 5124 (2014) (> 120 citations)
[2]L. C. Andreani and D. Gerace, PRB **73**, 235114 (2006) (> 170 citations)

Genetic optimization of PhCs: case of Si

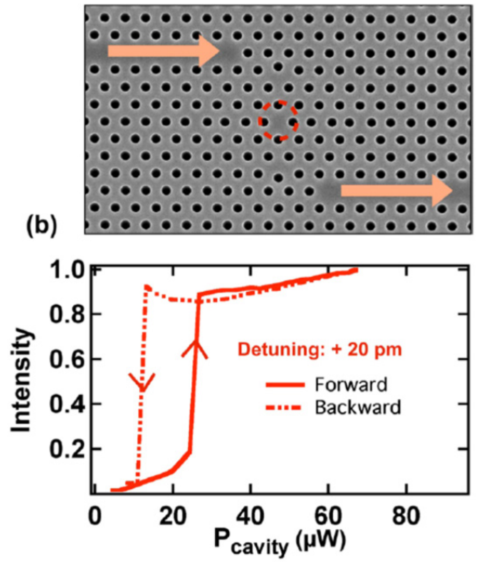
L3 photonic crystal slab nanocavity[1]





- Q value close to 2×10^6 for optimized cavities including imperfections
- $Q/V_m > 1 \times 10^6\ (\lambda/n)^{-3}$

H0 photonic crystal slab nanocavity[2]



- Q value close to 400000
- $Q/V_m > 1 \times 10^6\ (\lambda/n)^{-3}$
- Optical bistability with low threshold power

Access to optical nonlinearities thanks to high Q/V_m

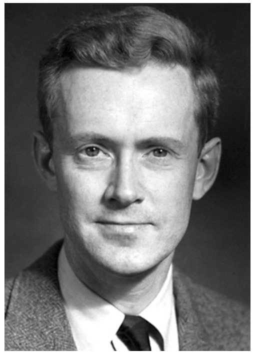
[1]Y. Lai *et al.*, APL **104**, 241101 (2014) (> 110 citations)

[2]U. P. Dharanipathy *et al.*, APL **105**, 101101 (2014)

Enhanced spontaneous emission rate: Purcell effect[1]

The Nobel Prize in Physics 1952

" "for their development of new methods for nuclear magnetic precision measurements and discoveries in connection therewith"



Edward Mills Purcell

Harvard University, Cambridge, MA, USA

Purcell factor giving the increase in the spontaneous emission rate:

$$F_p = \frac{3Q\lambda_c^3}{4\pi^2 n^3 V}$$

refractive index (for solid state)

B10. Spontaneous Emission Probabilities at Radio Frequencies. E. M. PURCELL, *Harvard University*.—For nuclear magnetic moment transitions at radio frequencies the probability of spontaneous emission, computed from

$$A_\nu = (8\pi\nu^2/c^3)h\nu(8\pi^3\mu^2/3h^2) \text{ sec.}^{-1},$$

is so small that this process is not effective in bringing a spin system into thermal equilibrium with its surroundings. At 300°K, for $\nu = 10^7$ sec.$^{-1}$, $\mu = 1$ nuclear magneton, the corresponding relaxation time would be 5×10^{21} seconds! However, for a system coupled to a resonant electrical circuit, the factor $8\pi\nu^2/c^3$ no longer gives correctly the number of radiation oscillators per unit volume, in unit frequency range, there being now *one* oscillator in the frequency range ν/Q associated with the circuit. The spontaneous emission probability is thereby increased, and the relaxation time reduced, by a factor $f = 3Q\lambda^3/4\pi^2 V$, where V is the volume of the resonator. If a is a dimension characteristic of the circuit so that $V\sim a^3$, and if δ is the skin-depth at frequency ν, $f\sim\lambda^3/a^2\delta$. For a non-resonant circuit $f\sim\lambda^3/a^3$, and for $a<\delta$ it can be shown that $f\sim\lambda^3/a\delta^2$. If small metallic particles, of diameter 10^{-3} cm are mixed with a nuclear-magnetic medium at room temperature, spontaneous emission should establish thermal equilibrium in a time of the order of minutes, for $\nu = 10^7$ sec.$^{-1}$.

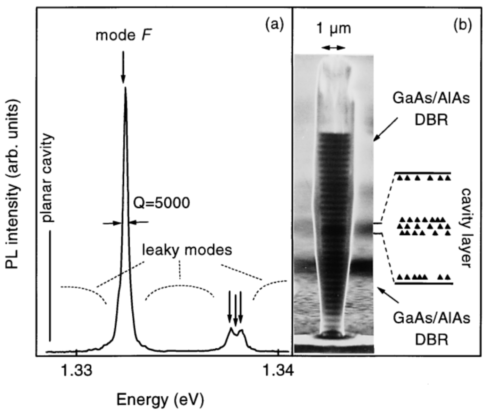
Article cited more than 3930 times (> 1600 times over the past 10 years)!

Tailoring the number of electromagnetic modes to which a quasimonochromatic dipole is coupled

[1]E. M. Purcell, Phys. Rev. **69**, 681 (1946).

Enhanced spontaneous emission rate: Purcell effect[1]

A first step toward cavity quantum electrodynamics (CQED)



Requirements (simplified treatment)
Localized dipole (wavelength λ_e, FWHM $\Delta\lambda_e$)
Single cavity mode (wavelength λ_c, FWHM $\Delta\lambda_c$)
and $\Delta\lambda_e \ll \Delta\lambda_c$

Since $\Delta\lambda_e \ll \Delta\lambda_c$ the escape time of spontaneous emission (SE) photons out of the cavity is much shorter than the radiative lifetime and reabsorption is negligible

$$F_P = \frac{3Q\lambda_c^3}{4\pi^2 n^3 V}$$

The total SE rate is given by F_P provided:

1. the emitter is on exact resonance,
2. the emitter is located at the antinode of the vacuum field,
3. its dipole is parallel to the vacuum electric field.

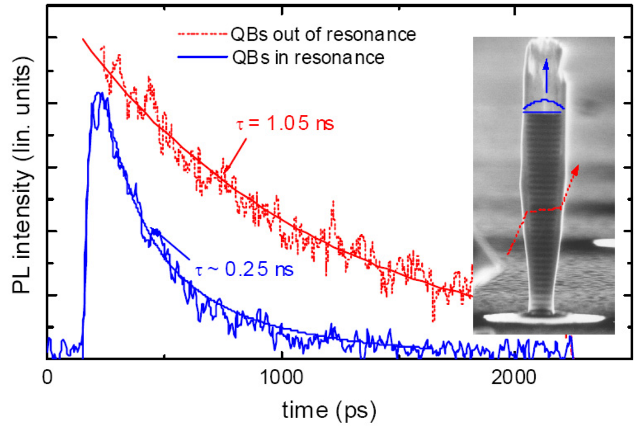
⇒ F_P is a figure of merit for the cavity alone since it describes its ability to increase the coupling of an ideal emitter with the vacuum field

[1]J.-M. Gérard *et al.*, Phys. Rev. Lett. **81**, 1110 (1998). (> 940 citations)

Enhanced spontaneous emission rate: Purcell effect[1]

A first step toward cavity quantum electrodynamics (CQED)

InAs/GaAs QDs in the core of a small volume high finesse micropillar



Global SE rate normalized to the "bulk" emitter SE rate $1/\tau_0$

modes are twofold polarization degenerate

electric field

emission into the leaky modes

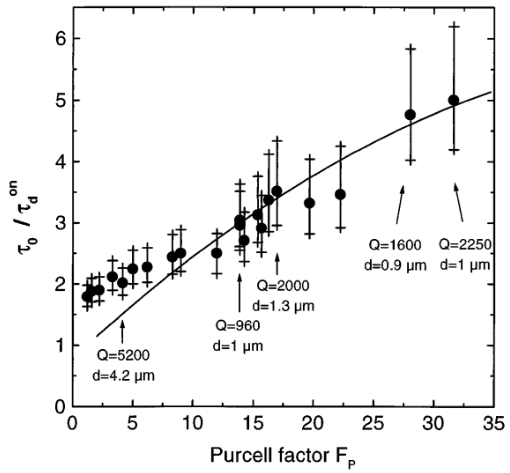
$$\frac{\tau_0}{\tau} = \frac{2F_p}{3}\frac{|\mathbf{E}(\mathbf{r})|^2}{|\mathbf{E}|^2_{max}}\frac{\Delta\lambda_c^2}{\Delta\lambda_c^2 + 4(\lambda_e - \lambda_c)^2} + 0.8$$

location of emitter vs cavity mode

emitter-cavity detuning

factor accounting for the random dipole orientation

≡ Experimental departure from F_P



Large QD number (~30) coupled to mode ⇒ statistical averaging of spatial and spectral distributions

Spontaneous emission rate enhancement by a factor up to 5

Promising for more efficient (brighter) single photon sources

Interest for quantum communications (more flying qubits)

[1] J.-M. Gérard *et al.*, Phys. Rev. Lett. **81**, 1110 (1998).

Indistinguishable single photon sources

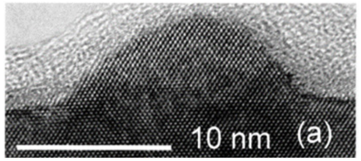
Source of "flying qubits" for quantum information

ARTICLES
PUBLISHED ONLINE: 7 MARCH 2016 | DOI: 10.1038/NPHOTON.2016.23
nature photonics

Near-optimal single-photon sources in the solid state

N. Somaschi[1†], V. Giesz[1†], L. De Santis[1,2†], J. C. Loredo[3], M. P. Almeida[3], G. Hornecker[4,5], S. L. Portalupi[1], T. Grange[4,5], C. Antón[1], J. Demory[1], C. Gómez[1], I. Sagnes[1], N. D. Lanzillotti-Kimura[1], A. Lemaître[1], A. Auffeves[4,5], A. G. White[3], L. Lanco[1,6] and P. Senellart[1,7]*

Nat. Phot. **10**, 340 (2016) (> 900 citations)

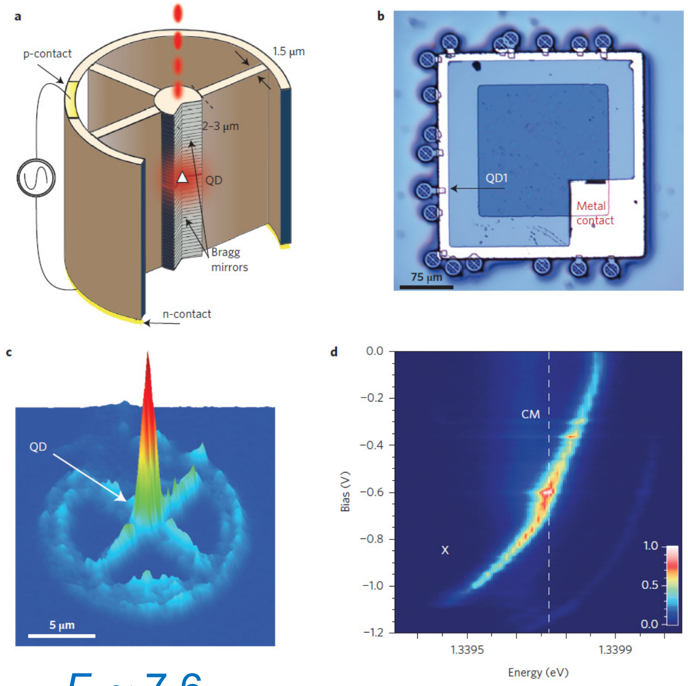


PRL **84**, 733 (2000)

- Under resonant excitation, $g^{(2)}(0) = 0.0028 \pm 0.0012$ together with an indistinguishability of 0.9956 ± 0.0045

Same λ, polarization, and spatial and temporal extent + Fourier-limited

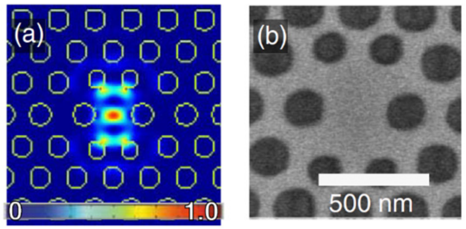
- Photon extraction efficiency of 65%
- Brightness 20 times larger than any source of equal quality

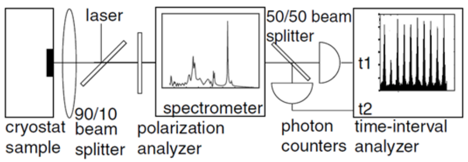


$F_P \sim 7.6$

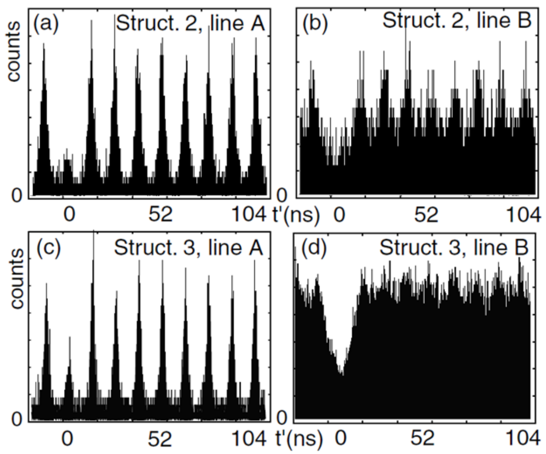
Enhanced spontaneous emission rate: Purcell effect[1]

QDs embedded in 2D-PhCs



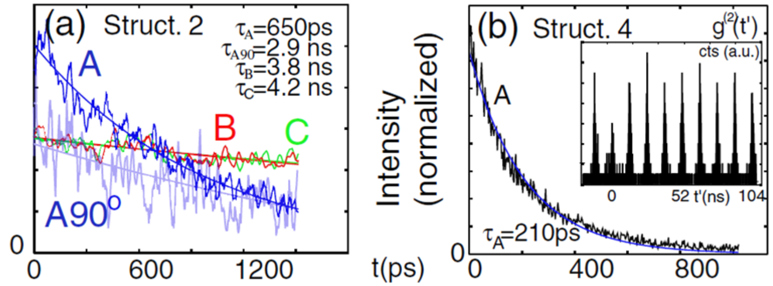


$g^{(2)}(0)$ measurements in the pulsed regime

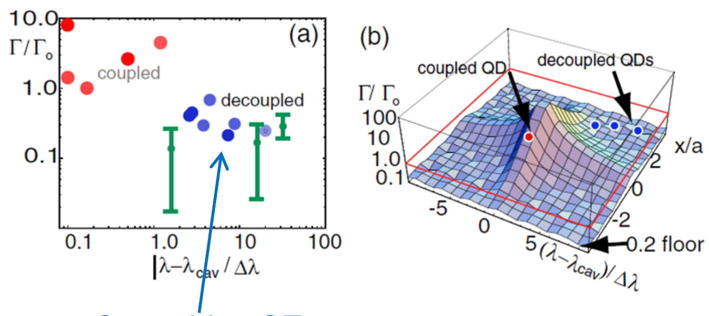


All QDs exhibit photon antibunching ($g^{(2)}(0) < 0.5$)

Exciton lifetime measurements



Spontaneous emission (SE) rate modification for QDs on- and off-resonance



Quenching SE rate

[1]D. Englund *et al.*, Phys. Rev. Lett. **95**, 013904 (2005). (> 810 citations)

Light-emitting diodes: historical account

History: first LED

A Note on Carborundum. ⇒ **SiC**

To the Editors of Electrical World:

SIRS:—During an investigation of the unsymmetrical passage of current through a contact of carborundum and other substances a curious phenomenon was noted. On applying a potential of 10 volts between two points on a crystal of carborundum, the crystal gave out a yellowish light. Only one or two specimens could be found which gave a bright glow on such a low voltage, but with 110 volts a large number could be found to glow. In some crystals only edges gave the light and others gave instead of a yellow light green, orange or blue. In all cases tested the glow appears to come from the negative pole, a bright blue-green spark appearing at the positive pole. In a single crystal, if contact is made near the center with the negative pole, and the positive pole is put in contact at any other place, only one section of the crystal will glow and that the same section wherever the positive pole is placed.

There seems to be some connection between the above effect and the e.m.f. produced by a junction of carborundum and another conductor when heated by a direct or alternating current; but the connection may be only secondary as an obvious explanation of the e.m.f. effect is the thermoelectric one. The writer would be glad of references to any published account of an investigation of this or any allied phenomena.

NEW YORK, N. Y. H. J. ROUND.

H. J. Round, Electrical World **49**, 309 (1907).

1907

Henry Joseph Round
(Marconi Co)

First LED operation

First LED operation (1906)

not understood until 1950's !

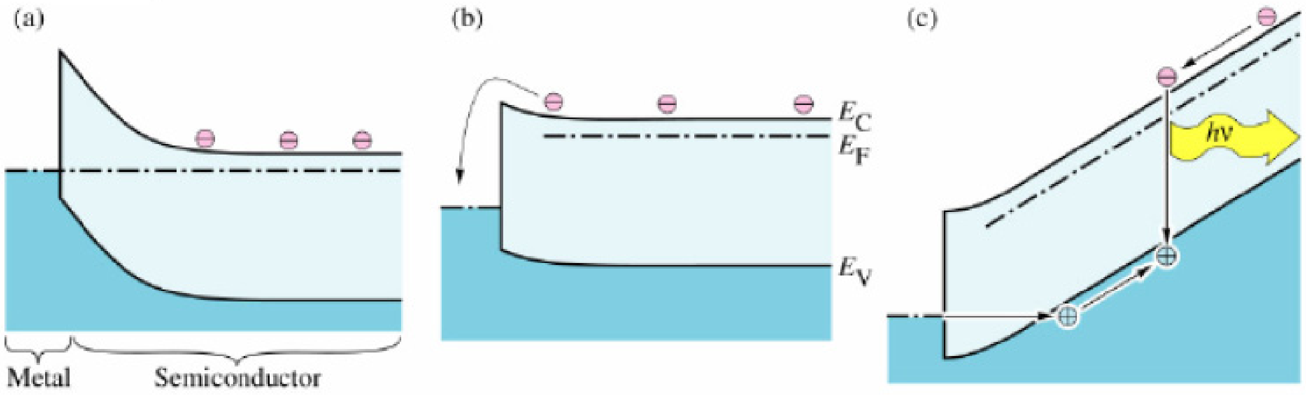


Fig. 1.2. Band diagram of Schottky diode under (a) equilibrium conditions, (b) forward bias, and (c) strong forward bias. Under strong forward bias, minority carrier injection occurs making possible near-bandgap light emission.

First LED did not operate with a PN junction !

History

- In 1961, **IR light emission from GaAs** – (Texas Instruments)
- **First visible (red) LED in 1962** – N. Holonyak, Jr. (General Electric)
- **First yellow and red-orange high-brightness LEDs** by George Craford (Hewlett-Packard) **in 1972**
- **First high-brightness blue LEDs** by Nakamura (Nichia, Japan) **in 1993**
- **First white LEDs** by Nakamura (Nichia, Japan) **in 1996**

Millennium Technology Prize for III-N LEDs and LDs

MILLENNIUM TECHNOLOGY PRIZE

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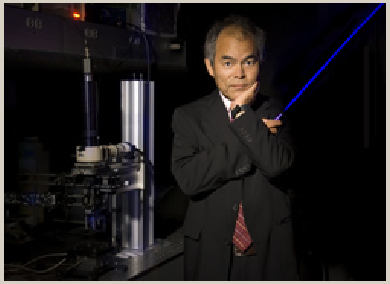
2004-2008 PRIZES

- 2008
- 2006
  - INNOVATION
  - APPLICATIONS
- 2004

WINNER: SHUJI NAKAMURA



Shuji Nakamura was born in 1954 in Japan on the island of Shikoku. He received his master's degree at the University of Tokushima in 1979.

He started his scientific and technological career outside mainstream Japanese technology, working as an engineer at Nichia Chemical, a small company working on phosphors (inorganic luminescent materials) in the countryside.

In Nichia Chemical's laboratory, with only a limited budget and modest support from company management, Nakamura developed the novel MOCVD technique. This enabled him to manufacture a bright-blue LED, which led in turn to a white LED and then to a blue laser.

In 1993 he stunned the optoelectronic community with the announcement of very-bright blue GaN-based light emitting diodes, LEDs. In rapid succession, he then announced a green GaN-based LED, a blue laser diode, and a white LED.

In 1994, Nakamura received his doctorate in engineering at the University of Tokushima. Five years later he left Japan to join the faculty of the University of California, Santa Barbara (UCSB). At UCSB he has built up a significant research programme in new areas of nitride research.

Further Reading

Article on Nakamura in Scientific American, 5 July, 2000

BBC World September 2006

Nobel Prize in Physics 2014 for blue LEDs

The Nobel Prize in Physics 2014
Isamu Akasaki, Hiroshi Amano, Shuji Nakamura

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The Nobel Prize in Physics 2014

Photo: A. Mahmoud
Isamu Akasaki
Prize share: 1/3

Photo: A. Mahmoud
Hiroshi Amano
Prize share: 1/3

Photo: A. Mahmoud
Shuji Nakamura
Prize share: 1/3

The Nobel Prize in Physics 2014 was awarded jointly to Isamu Akasaki, Hiroshi Amano and Shuji Nakamura *"for the invention of efficient blue light-emitting diodes which has enabled bright and energy-saving white light sources"*.